

UCRHEP-T436  
September 2007

# **Z<sub>3</sub> Dark Matter and Two-Loop Neutrino Mass**

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## **Abstract**

Dark matter is usually distinguished from ordinary matter by an odd-even parity, i.e. the discrete symmetry  $Z_2$ . The new idea of  $Z_3$  dark matter is proposed with a special application to generating radiative Majorana neutrino masses in two-loop order.

Neutrinos have mass [1] and the Universe has dark matter [2]. Whereas the nearly massless neutrinos themselves could only account for a very small fraction of the latter, the mechanism by which they acquire mass may involve particles which do form the bulk of the dark matter itself. A recent realistic proposal [3, 4, 5, 6, 7, 8, 9] is to add a second scalar doublet  $\eta = (\eta^+, \eta^0)$  to the Standard Model (SM) of particle interactions together with three singlet neutral Majorana fermions  $N_i$ , such that  $\eta$  and  $N_i$  are odd under an exactly conserved  $Z_2$  discrete symmetry whereas all SM particles are even. In the presence of the allowed quartic scalar interaction

$$\frac{1}{2}\lambda_5(\Phi^\dagger\eta)^2 + H.c., \quad (1)$$

where  $\Phi = (\phi^+, \phi^0)$  is the SM Higgs doublet,  $\eta^0$  is split so that  $\eta_R^0 = \sqrt{2}Re(\eta^0)$  and  $\eta_I^0 = \sqrt{2}Im(\eta^0)$  have different masses, resulting simultaneously in (A) the one-loop radiative generation (Fig. 1) of Majorana neutrino masses through the allowed interaction

$$h_{ij}(\nu_i\eta^0 - l_i\eta^+)N_j + H.c., \quad (2)$$

and (B) the possible identification [3, 10, 11, 12] of  $\eta_R^0$  or  $\eta_I^0$  as dark matter. The collider phenomenology of  $\eta$  has also been discussed [10, 13].

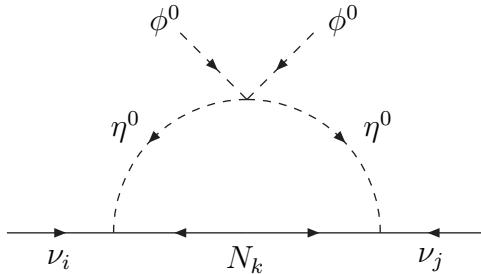


Figure 1: One-loop generation of neutrino mass.

The simple idea that  $\eta_R^0$  or  $\eta_I^0$  is absolutely stable because of the  $Z_2$  discrete symmetry goes back 30 years [14]. On the other hand, the simplest possible realization of dark matter

is to postulate [15] a real scalar field  $D$  which is odd under  $Z_2$ . The SM Higgs potential is then extended to

$$V = m_1^2 \Phi^\dagger \Phi + \frac{1}{2} m_2^2 D^2 + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{8} \lambda_2 D^4 + \frac{1}{2} \lambda_3 D^2 (\Phi^\dagger \Phi). \quad (3)$$

In the presence of electroweak symmetry breaking,  $\Phi^\dagger \Phi$  is replaced by  $(v + h)^2/2$ , where  $m_1^2 + \lambda_1 v^2/2 = 0$ , hence

$$V = \frac{1}{2} \lambda_1 v^2 h^2 + \frac{1}{2} (m_2^2 + \frac{1}{2} \lambda_3 v^2) D^2 + \frac{1}{2} \lambda_1 v h^3 + \frac{1}{8} \lambda_1 h^4 + \frac{1}{8} \lambda_2 D^4 + \frac{1}{2} \lambda_3 v h D^2 + \frac{1}{4} \lambda_3 h^2 D^2, \quad (4)$$

where  $h$  is the SM Higgs boson. The phenomenology of this simplest of all dark-matter scenarios has recently been updated [16, 17]. Of particular interest is the decay  $h \rightarrow DD$  which is invisible, thus allowing [13]  $m_h$  to be below the present bound of 114.4 GeV from LEP data [18].

In most studies however, dark matter is synonymous with the lightest neutralino in  $R$ -parity conserving supersymmetry [2] which again is based on a  $Z_2$  discrete symmetry. In fact, the choice of  $Z_2$  for dark matter is universally practiced, but is not required by any fundamental principle; it is just the simplest hypothesis which works. Consider thus the new idea of  $Z_3$  dark matter. How does it work? and what are its implications?

Let  $\chi$  be a neutral complex scalar singlet, then instead of Eq. (3), consider

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \chi^\dagger \chi + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\chi^\dagger \chi) (\Phi^\dagger \Phi) + \frac{1}{6} \mu \chi^3 + \frac{1}{6} \mu^* (\chi^\dagger)^3. \quad (5)$$

This extended Higgs potential is invariant under  $Z_3$  with  $\chi$  transforming as  $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$ , and  $\mu$  may be chosen real by absorbing its phase into  $\chi$ . With the replacement  $\Phi^\dagger \Phi = (v + h)^2/2$ , this becomes

$$\begin{aligned} V = & \frac{1}{2} \lambda_1 v^2 h^2 + (m_2^2 + \frac{1}{2} \lambda_3 v^2) \chi^\dagger \chi + \frac{1}{2} \lambda_1 v h^3 + \frac{1}{8} \lambda_1 h^4 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 \\ & + \lambda_3 v h (\chi^\dagger \chi) + \frac{1}{2} \lambda_3 h^2 (\chi^\dagger \chi) + \frac{1}{6} \mu [\chi^3 + (\chi^\dagger)^3]. \end{aligned} \quad (6)$$

Comparing this with Eq. (4), it is clear that  $\chi$  is as good a dark-matter candidate as  $D$ , and indistinguishable from  $D$  unless the cubic self-interaction  $\chi^3$  can be established. This illustrates the generic possibility that dark matter may be distinguished from ordinary matter by a symmetry larger than  $Z_2$  and yet not be discovered easily in experiments.

Recalling that dark matter may be related to radiative neutrino mass [3], it is shown below exactly how  $\chi$  enables just such a scenario in two loops. There have been two basic two-loop mechanisms of radiative neutrino mass [19]: one via the exchange of two  $W$  bosons [20, 21, 22, 23, 24], the other via a new trilinear scalar interaction [25, 26, 27, 28, 29, 30, 31, 32]. Both have SM charged leptons in the loop and the new particles involved cannot be dark matter. In a three-loop extension [33, 34], the innermost loop may be populated with dark-matter candidates, but the constraint due to flavor changing radiative decays such as  $\mu \rightarrow e\gamma$  may not be so easily satisfied [4]. In the context of the supersymmetric  $E_6/U(1)_N$  model [35], a two-loop mechanism with  $Z_2$  dark matter has also been proposed recently [8], with the novel feature that the  $(\lambda_5/2)(\Phi^\dagger\eta)^2$  interaction is generated in one loop.

In the case of  $Z_3$  dark matter, consider the following additions to the SM:

$$\text{scalars : } \chi_{1,2,3} \sim \omega, \quad (7)$$

$$\text{fermions : } (N, E)_{L,R}, S_{L,R} \sim \omega, \quad (8)$$

where  $\chi_{1,2,3}$ ,  $N$  and  $S$  are neutral and  $E$  has charge  $-1$ . Using the allowed interactions

$$\begin{aligned} \mathcal{L}_{int} = & h_{ij}(\bar{N}_R\nu_{iL} + \bar{E}_Rl_{iL})\chi_j + f_1\bar{S}_L(N_R\phi^0 - E_R\phi^+) + f_2\bar{S}_R(N_L\phi^0 - E_L\phi^+) \\ & - \frac{1}{2}f_{3i}\chi_iS_LS_L - \frac{1}{2}f_{4i}\chi_iS_RS_R - \frac{1}{6}\mu_{ijk}\chi_i\chi_j\chi_k + H.c. \end{aligned} \quad (9)$$

and the allowed Dirac masses  $m_N, m_E, m_S$ , the following two-loop diagram (Fig. 2) for neutrino mass is generated. Lepton number is now conserved multiplicatively with all fields even except  $\nu$ ,  $l$ ,  $N$ ,  $E$ , and  $S$  which are odd. Note that  $Z_3$  is tailor-made for such a

mechanism because of the trilinear scalar interaction  $\chi^3$ . Note also that it is an explicit two-loop realization of the unique dimension-five operator  $\nu_i \nu_j \phi^0 \phi^0$  [36, 37].

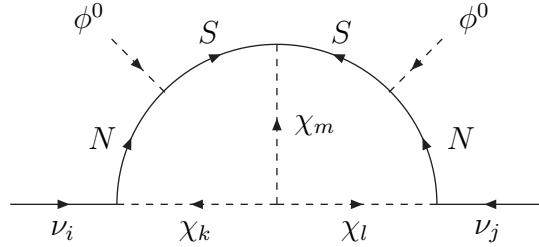


Figure 2: Two-loop generation of neutrino mass.

The neutrino mass matrix is then approximately given by

$$(\mathcal{M}_\nu)_{ij} = \frac{v^2}{512\pi^4} \sum_{k,l,m} h_{ik} h_{jl} \mu_{klm} \left[ \frac{f_1^2 f_{3m}}{(M_{eff})^2} + \frac{f_2^2 f_{4m} m_N^2}{(M'_{eff})^4} \right]. \quad (10)$$

For illustration, let  $h = 0.003$ ,  $f = 0.36$ ,  $\mu = 100$  GeV and  $M = M' = m_N = 1$  TeV, then neutrino masses are of order 0.1 eV, and flavor changing radiative decays such as  $\mu \rightarrow e\gamma$  (which depend crucially on  $h$ ) are well below experimental bounds [29]. [On the other hand, there is enough freedom in the choice of the above couplings to allow them to be observable in the near future as well.]

Let  $\chi_1$  be the lightest of  $\chi_{1,2,3}$ , then it is a suitable dark-matter candidate in the same way as the  $Z_2$  candidate  $D$  [16]. In addition,  $\chi_i$  may be discovered through the decay

$$E \rightarrow l_i \chi_j \quad (11)$$

and  $\chi_2 \rightarrow \chi_1 l_i^+ l_j^-$ , etc. Another important feature of this model is the mixing between  $S$  and  $N$  through  $\langle \phi^0 \rangle = v/\sqrt{2}$ . This allows the decay of the heavier mass eigenstate  $N_2$  into the lighter  $N_1$ , i.e.

$$N_2 \rightarrow N_1 Z, \quad (12)$$

and  $N_1 \rightarrow \nu_i \chi_j$ . The decay  $E \rightarrow N_1 W^-$  may also be possible. In searching for the SM Higgs boson  $h$ , the invisible decay  $h \rightarrow \chi_1 \chi_1$  and the more generic  $h \rightarrow \chi_i \chi_j$  should also be kept in mind.

A possible variant of this model is to substitute the particles of Eqs. (7) and (8) with

$$\text{scalars : } \chi, (\eta^+, \eta^0) \sim \omega, \quad (13)$$

$$\text{fermions : } (N_{1,2,3})_{L,R} \sim \omega, \quad (14)$$

where  $N$  is neutral. In this case, the two-loop diagram for neutrino mass (Fig. 3) involves necessarily the mixing of  $\chi$  and  $\eta^0$ . This renders them unsuitable as dark-matter candidates because each eigenstate must couple to the  $Z$  boson (in the same way as the scalar neutrino in supersymmetry) and thus ruled out by direct-search experiments [2]. In the case of  $Z_2$  dark matter,  $\eta^0$  is allowed to be split so that  $\eta_R^0$  and  $\eta_L^0$  have different masses, thereby evading the above constraint. Here this is not possible because of the  $Z_3$  symmetry.

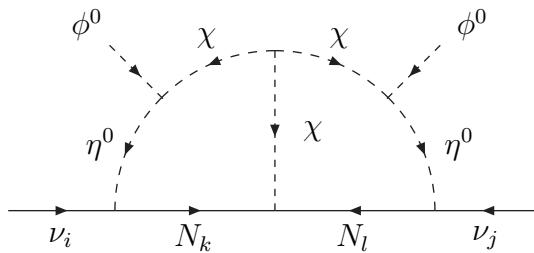


Figure 3: Another two-loop generation of neutrino mass.

The lightest  $N_i$  may be considered [3, 4, 38] as the dark-matter candidate in this case, but because its relic abundance depends on its interaction with charged leptons, flavor-changing radiative decays such as  $\mu \rightarrow e\gamma$  are difficult to suppress. One solution [9] is to add a singlet scalar which is also trivial under  $Z_3$  so that  $N\bar{N} \rightarrow hh$  occurs at tree level.

In  $R$ -parity conserving supersymmetry, the dark-matter candidate, i.e. the lightest neutralino, is just one of an entire class of new particles to be discovered which are odd under  $Z_2$ . In the context of  $Z_3$  dark matter, this may also be the case. If they have  $SU(3)_C$  interactions, they will also be produced abundantly at the LHC and have a good chance of being discovered in the near future.

Another possible avenue of thought is that dark matter may be a hint that there could be new particles in a hidden sector which communicate with ordinary particles only through the Higgs sector. This makes sense also in the context of the Minimal Supersymmetric Standard Model, where the only allowed bilinear term of the superpotential is  $\mu H_u H_d$ . If  $\mu$  is replaced by a singlet superfield, the latter may serve as the link to a completely new world of particles, perhaps with its own gauge group and conservation laws. There may even be several such links.

In conclusion, it has been pointed out that the universal assumption of  $Z_2$  dark matter is not the only available option. A specific  $Z_3$  alternative is proposed which coincides with the new notion that dark matter is responsible for neutrino mass, generating the latter radiatively in two loops. The dark-matter candidate here is a complex singlet scalar field which interacts with ordinary matter mostly through the canonical Higgs boson of the Standard Model.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

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